## QUIZ 18 SOLUTIONS: LESSONS 24-25 OCTOBER 31, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. Consider the following table providing information about the function $f(x, y)$ :

| $(x, y)$ | $f(x, y)$ | $f_{x}(x, y)$ | $f_{y}(x, y)$ | $f_{x x}(x, y)$ | $f_{y y}(x, y)$ | $f_{x y}(x, y)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(-2,1)$ | 7 | -8 | 4 | 4 | 2 | -4 |
| $(0,0)$ | 0 | 0 | -2 | 2 | 2 | 0 |
| $(2,-1)$ | 3 | 0 | 0 | 0 | 2 | 4 |

(a) $[1 \mathrm{pt}]$ Which of the above $(x, y)$ is a critical point of $f$ ?

Solution: The critical point is

$$
(x, y)=(2,-1)
$$

because $f_{x}(2,-1)=0=f_{y}(2,-1)$.
(b) $[1 \mathrm{pt}]$ Is the $(x, y)$ from part (a) a minimum, maximum, saddle point, or inconclusive?

Solution: $(x, y)=(2,-1)$ is a saddle point.
(c) $[2 \mathrm{pts}]$ Give a mathematical reason for you answer to part (b).

Solution: $(x, y)=(2,-1)$ is a saddle point because
$D(2,-1)=f_{x x}(2,-1) f_{y y}(2,-1)-\left(f_{x y}(2,-1)\right)^{2}=0(2)-(4)^{2}=-16$ is less than 0 .
2. [6 pts] Let

$$
f(x, y)=3 x^{2} y
$$

If we want to maximize $f$ subject to

$$
3 x+2 y^{2}=6
$$

using the method of LaGrange multipliers, our system of equations would be

$$
\begin{aligned}
6 x y & =\lambda \sqrt{3} \\
3 x^{2} & =\lambda \sqrt[4 y]{3} \\
3 x+2 y^{3} & =6
\end{aligned}
$$

(Fill in the green boxes.)

